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## Final Technical Report for AFOSR contract 89NM605

During the reporting period (94/95) very good progress has been made on the projects covered by this contract.

Collaboration has been continued with Kurt Bryan related to crack determination. Following up on our earlier results concerning the determination of multiple cracks from boundary measurements we have completed a study of the relation between the electrical boundary data coming from a cluster of small cracks and that coming from a single effective macroscopic crack [1]. As a simple example, consider a two dimensional array of cracks lying one after the other on a family of closely spaced parallel lines. In this case our analysis shows that if the "in line" distance between successive cracks is exponentially small then the data looks like that coming from a single macroscopic crack. If not, then the data may be used to distinguish the cracks! In addition to being an interesting "homogenization"-result, this also provides an immediate explanation why closely spaced "in line" cracks are quite accurately reconstructed by the numerical algorithm we have designed.

I gave a one hour talk about the work on crack detection at the CBMS-NSF conference on Nondestructive Evaluation and Inverse Problems, in Lexington in June, 1995, as well as a plenary talk at the IFIP conference on Modelling and Optimization of Distributed Parameter Systems with Applications to Engineering, in Warsaw, Poland, in July 1995. In connection with my talk at this latter conference I have almost completed a survey article of the work on crack detection [2].

Jointly with a graduate student (Donna Fengya) I have studied the effect of small inclusions of different conductivity inside a homogenous conductor [3]. We have derived an asymptotic representation formula for the voltage potential (including a convergence proof) and we have used this representation formula to establish Lipschitz estimates for the locations and the relative sizes of the inclusions. On one hand this work significantly extends work which Avner Friedman and I carried out earlier concerning inclusions with zero or infinite conductivity, on the other hand this work has a significant computational component, which demonstrates the practical viability of using electrical measurements to find such small inhomogenities. This study is also very much related to our work to demonstrate the viability of impedance imaging for determining corrosion effects. In this context we have made good progress on a study of the effective current boundary condition for a metal component with surface corrosion pits, covered by a dilute solution; this study has involved a considerable search of the corrosion engineering literature. A first part of this study is already reported in a paper with J.-M. Xu [4]. In this paper we examine the questions of existence and uniqueness of solutions for a model (of the dilute solution) based on the so-called Butler-Volmer boundary condition. We put particular emphasis on the regime where the boundary is in a transition to passivity (due to the fact that it is largely covered by an oxide layer) – this is practically and mathematically a very interesting regime. It is worth mentioning that we have developed contacts with a material science group at the EPFL (headed by D. Landolt). We hope these contacts will lead to

experimental work concerning the transition to passivity. In a second, forthcoming paper [5] we have by means of a multiple scale analysis derived effective boundary conditions for the metal component, based on the assumption that the dilute solution is very shallow. Using this effective boundary condition we have also carefully examined the question of identifiability of the corrosion damage. Part of the study of corrosion imaging has been related to plates – we have for instance developed (and studied the performance) of a numerical algorithm to image corrosion damage on one side of a plate (the interior) from measurements of voltage and current data on the opposite side [6]. For a mathematically vaguely related problem we have developed a method to reconstruct the interface between the electrolytic bath and the aluminum in a so-called Hall-Héroult cell (a standard tool to produce aluminum). This cell is also “thin in one direction” (it is very shallow); for this problem we establish an estimate of the error between the interface reconstructed from the “shallow” model and the “true” interface (corresponding to the higher dimensional problem) and we also study higher order approximations [7]. The electrostatic data is known on both top and bottom of the cell. My collaborator on this project (Michel Romerio of the EPFL) has strong ties (as a consultant) to Alusuisse.

The book project, [8], with John Sylvester and Gunther Uhlmann (entitled: Electrical Impedance Tomography) has progressed steadily, although somewhat slower than we had initially hoped. One of the reasons for the delay is our decision to include a section with the very recent results about two dimensional uniqueness (due to A. Nachman) as well as a section (somewhat longer than originally envisaged) dealing with the most recent results about crack detection. We expect to have a complete manuscript finished in the spring of 1996. With John and Gunther I have also organized an AMS summer conference on Electrical Impedance Imaging, which took place in Seattle in July 1995.

The work on first order corrections to homogenized eigenvalues has continued successfully. We study the possible limits of  $(\lambda^\epsilon - \lambda)/\epsilon$  as  $\epsilon \rightarrow 0$ . Here  $\lambda^\epsilon$  is an eigenvalue of a periodic, composite medium with material parameters  $a(x/\epsilon)$  ( $\epsilon$  is the microscopic length scale) and  $\lambda$  is the corresponding eigenvalue of the effective medium with constant material parameters,  $A$ . We call these limits  $(\lambda_*)$  first order corrections, since they give rise to asymptotic formulae  $\lambda^\epsilon = \lambda + \epsilon\lambda_* + o(\epsilon)$ . Jointly with a graduate student (Shari Moskow) I have succeeded in proving that for a polygonal domain whose sides have rational slopes (relative to the periodic microstructure) there is generally not just one correction but frequently a continuum of possible corrections [9], [10]. For the case of Dirichlet boundary conditions this result had already been established during the last reporting period – much to our surprise we found (during this reporting period) that the same result holds for Neumann boundary conditions.

We provide a very explicit representation formulas for the set of possible first order corrections. Whether a limit (of  $(\lambda^{\epsilon_n} - \lambda)/\epsilon_n$ ) is attained for a particular sequence  $\epsilon_n \rightarrow 0$ , and exactly what is the value of this limit, depends on the interaction between the periodic microstructure and the macroscopic boundary,  $\partial\Omega$ . Our analysis, which is quite

intricate, involves a combination of error-estimates for approximation of eigenvalues, a careful study of the minimal smoothness properties required to derive higher order estimates of homogenization convergence as well as a very detailed analysis of the associated boundary layers. In the case of Neumann boundary conditions we also make extensive use of duality. The techniques developed have immediate implications beyond the eigenvalue problem, for instance for the boundary layers that inevitably appear in connection with many homogenization problems. It is exactly because of this close link to boundary layers that we were initially inclined to think that Neumann boundary conditions would lead to behavior significantly different from that exhibited with Dirichlet boundary conditions (they do in one dimension – but not in two dimensions!) Some of results we have proven were conjectured in earlier work with Fadil Santosa . We are currently in the process of carrying out a number of two dimensional calculations of the first order eigenvalue corrections and the true eigenvalues (with oscillatory material parameters). This work (which will be contained in [11]) represents a significant extension of our earlier calculations.

My investigation concerning certain elliptic inverse problems, originating from Tokamak fusion, has continued. In its simplest form this investigation concerns the two dimensional Laplace operator. The problem is then to identify  $f \geq 0$  of the equation  $\Delta u + f(u) = 0$  in  $\Omega$ , from knowledge of the boundary flux  $\frac{\partial}{\partial \nu} u$ , corresponding to homogeneous Dirichlet data. Extending our previous work, Elena Beretta and I have succeeded in proving that if the domain has a proper corner (with angle different from  $\pi$ ) and if it is apriori known that  $f(0) = 0$ , then it is possible to recover all the derivatives of  $f$  at 0 from knowledge of  $\frac{\partial}{\partial \nu} u|_{\partial \Omega}$ , [12]. If it is *not* known that  $f(0) = 0$  then the same result applies, except we have to exclude a possibly countable number of angles  $\theta < \pi/2$ . We believe this latter exclusion is for technical reasons alone, and that the recovery should still be possible in the case of these angles. It is not that difficult to see that recovery of  $f(0)$  and  $f'(0)$  is always possible if there is a proper corner. Indeed a relatively straightforward application of the hodograph transformation yields that if recovery of  $f(0)$  is impossible (and  $f$  is  $C^\infty$ ) then the domain must have a  $C^\infty$  boundary. We suspect that if recovery of any of the derivatives of  $f$  at 0 is impossible, then the domain must indeed be a disk, but we have so far been unable to prove this in any generality. I have recently been able to establish some results to this effect for affine  $f$ . It is well known that corners (of angle  $\pi/2$ ) occur in plasmas of Tokamaks based on the so-called divertor technology. We are currently, using methods of hodograph transformation, in the process of examining the reconstruction of  $f$  for domains that are perturbations of disks and we have obtained some encouraging partial results. I gave a plenary talk about this work at the annual meeting of the SIAM Activities Group for Control and Systems Theory, in St Louis in April 1995. A paper, surveying this work, is also in preparation [13].

During the reporting period I have given invited talks at several universities, for instance at Texas A& M, University of Texas Austin, Cornell University, University of Maryland Baltimore County, University of Maryland College Park, Ecole Polytechnique

Federale de Lausanne, Aalborg University, Aarhus University, Universität Göttingen and Eidgenössische Technische Hochschule Zürich.

- [1] Effective behaviour of clusters of microscopic cracks inside a homogeneous conductor. Submitted to *Asymptotic Analysis*. (with K. Bryan).
- [2] Crack detection by Electrical Impedance Imaging. A survey of results. To appear, *Control and Cybernetics*.
- [3] Identification of small conductivity imperfections by boundary measurements. Continuous dependence and computational reconstruction. In preparation (with D. Fengya)
- [4] A nonlinear elliptic boundary value problem related to corrosion modeling. Submitted to *Quarterly of Applied Math.* (with J.-M. Xu).
- [5] The derivation of an effective nonlinear boundary condition for a corroding surface. Applications to electrostatic imaging of the damage. In preparation (with Fadil Santosa and J.-M. Xu).
- [6] A method for imaging corrosion damage in thin plates from electrostatic data. To appear, *Inverse Problems*. (with Peter Kaup and Fadil Santosa).
- [7] Determination of the interface between the fluids in a Hall-Héroult cell from measurements of electric potentials and currents on the electrodes. In preparation (with M. Romerio).
- [8] *Electrical Impedance Tomography*, book in preparation (with J. Sylvester and G. Uhlmann).
- [9] First order corrections to the homogenized eigenvalues of a periodic composite medium. A convergence proof. To appear, *Proc. Roy. Soc. Edinburgh*. (with S. Moskow).
- [10] First order corrections to the homogenized eigenvalues of a periodic composite medium. The case of Neumann boundary conditions. Submitted to *Indiana Univ. Math J.* (with S. Moskow).
- [11] First order corrections to the homogenized eigenvalues of a periodic composite medium. Two dimensional computational examples. In preparation (with S. Moskow and F. Santosa).
- [12] An inverse problem originating from magnetohydrodynamics III. Domains with corners of arbitrary angles. *Asymptotic Analysis*, **11** (1995), pp. 289–315. (with E. Beretta).
- [13] An inverse problem originating from magnetohydrodynamics. A survey of results. To appear in proceedings of College de France.